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Determination of elastic constants of anisotropic solids from elastodynamic Green's functions

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Abstract

This paper describes a number of elastodynamic experiments that have been used to measure the elastic constants of anisotropic solids, or potentially could be used for that purpose. The inversion algorithms that are used to recover the elastic constants from measured data are discussed.

Keywords: Elastic constants; Anisotropic solids; Green's functions

1. Introduction

Recently a number of measurements of elastic constants of anisotropic solids have been reported that are based on point excitation and detection and interpreted in terms of elastodynamic Green's functions. Among the techniques employed are laser excitation and laser interferometric detection, small aperture piezoelectric transducers for excitation and detection, capillary and pencil lead fracture generation, capacitive detection and scanning acoustic microscopy. The aim of this paper is to review these elastodynamic methods and to describe the forward and inversion algorithms that are used.

2. Time domain methods

Time domain experiments involve applying a force with a step-function or δ-function time dependence at a point in a solid, and then measuring the time dependent displacement at some other point. For a force with step function time dependence

$$F(t)=\Theta(t)=\begin{cases}0, & t<0,\\ 1, & t>0,\end{cases} \tag{1}$$

applied to an infinite elastic continuum, the response or Greens's function $G_{sp}(\mathbf{x},t)$ can be expressed in the form [1]

$$G_{sp}(\mathbf{x},t)=\sum_n\left\{\frac{-1}{8\pi^2\rho}\int_\Omega d\Omega s^{(n)3}\Lambda_{sp}^{(n)}\delta(t-\mathbf{s}^{(n)}\cdot\mathbf{x}) + \frac{\Theta(t)}{8\pi^2\rho x}\int_0^{2\pi}d\phi s^{(n)2}\Lambda_{sp}^{(n)}\right\}. \tag{2}$$

The sum with respect to n is taken over the three acoustic branches. The first integral is taken over the unit hemisphere centred on the observation direction, with $d\Omega$ denoting the solid angle element in which the slowness vector lies. The second term is a line integral taken over the periphery of this hemisphere. $G_{sp}(\mathbf{x},t)$ depends, through the factor $\Lambda_{sp}^{(n)}=U_s^{(n)}U_p^{(n)}$, on the projection of the polarization vector $\mathbf{U}^{(n)}$ of each mode on the forcing and sensing directions. Calculated Green's functions show wide variation in shape, depending on the elastic constants, direction of observation point with respect to source point, and the components of the force and displacement referred to.

One of the striking features of these waveforms are the singular features they contain – discontinuities and kinks etc. These are known as wave arrivals. These singularities propagate outwards from the source at the group velocities in each direction, and thus lie on the wave surface, i.e. the group velocity surface scaled by a factor t. There are separate sheets for the individual

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branches (L, FT, ST), the inner two of which are often folded in a complicated way. For an elastic half space there are also head waves and, along the surface, there is the Rayleigh pole, which lags behind the slowest T wavefront, and which, along the surface, propagates the dominant singularity. For a plate there are also numerous multipass wave arrivals.

In some experiments, it is only the singularities in the waveform that can be clearly distinguished, the remaining features being obscured by transducer ringing and other experimental artifacts. Every and Sachse [2] have implemented the following strategy for recovering elastic constants $C_{\alpha\beta}$ from a set of measured group velocities, **V**, associated with wave arrivals. With a starting set of $C_{\alpha\beta}$, **V**s having the directions but not in general the magnitudes of the measured **V**s are calculated numerically. A succession of progressively improving values of the $C_{\alpha\beta}$ are then generated which, at each step, reduce the mean square difference between the measured and calculated **V**s.

For data pertaining to certain high symmetry directions such as 4-fold axes in cubic and tetragonal crystals, it is possible to derive closed form expressions relating the group velocities to the $C_{\alpha\beta}$, not only for the modes whose **n** lie in the symmetry direction (for which the phase and group velocities coincide), but also for the so-called oblique modes whose **n** lie away from the axis in symmetry planes. These expressions allow the $C_{\alpha\beta}$ to be obtained by analytic means [3].

Some techniques, such as capillary fracture generation coupled with capacitive detection yield clear, well defined waveforms that can fruitfully be used to recover elastic-constants. We have determined the elastic constants C_{12} and C_{44} of a (001)-oriented Si crystal by fitting the calculated infinite continuum Green's function to single epicentral and near-epicentral waveforms.

Chai and Wu [4] have measured the directional dependence of the group velocities of laser generated surface acoustic wave (SAWs) in the (111) surface of silicon and in a unidirectional fibre composite, and have used a simplex optimization method to recover the elastic constants. Their algorithm is very similar to that of Every and Sachse [2] for bulk waves.

3. Frequency domain methods

It is common practice in frequency domain experiments to image the displacement response over a surface or, in the case of SAWs, to record the SAW amplitude as a function of direction in the surface. In both cases, one of the most striking observations is that of energy flux focusing [5]. For bulk waves, it can be shown that, in the far field limit, the intensity I is inversely proportional to the Gaussian curvature of the slowness surface. Lines of zero curvature thus map onto caustics where I is mathematically infinite. These lines coincide in direction with folds in the wave surface. These effects have been extensively studied with thermal phonon imaging at liquid helium temperatures [5]. Focusing patterns depend on the elastic constant ratios, and there have been occasional efforts to use this to 'fine tune' elastic constants [6,7].

Going beyond the ray approximation, for a point force with sinusoidal time dependence of frequency ω acting at the origin in an infinite elastic continuum, the displacement amplitude at point **x** is given by the Green's function [1]

$$\tilde{G}_{sp}(\mathbf{x},\omega)=\sum_n\left\{\frac{\mathrm{i}\omega}{8\pi^2\rho}\int_\cap \mathrm{d}\Omega s^{(n)3}\Lambda_{sp}^{(n)}\,\mathrm{e}^{\mathrm{i}\omega\mathbf{s}^{(n)}\cdot\mathbf{x}}+\frac{1}{8\pi^2\rho x}\int_0^{2\pi}\mathrm{d}\phi s^{(n)2}\Lambda_{sp}^{(n)}\right\}. \quad (3)$$

For finite ωx the focusing caustics unfold into Airy, Pearcey and higher order diffraction patterns. The broad overall features of the focusing pattern survive, but the fine structure of the caustics is lost and in its place there is the expected pattern of diffraction fringes whose spacing becomes broader as the frequency is lowered. This effect is clearly evident in the acoustic microscopy images of anisotropic solids obtained by Weaver et al. [8] and others. The main features of their images are in good agreement with the computed Green's function $\tilde{G}_{33}(\mathbf{x},\omega)$ corresponding to the experimental parameters. Wuerz et al. [9] have used acoustic microscopy to determine the three elastic constants of GaAs, making use of a simplex inversion algorithm.

SAWs also show focusing due to the variable curvature of the SAW slowness curve, with points of inflection mapping onto cusps in the group velocity curve and focusing caustics. These effects have been discussed in a number of papers (for references see [10]) and they potentially provide another means of determining elastic constants.

References

[1] A.G. Every and K.Y. Kim, J. Acoust. Soc. Am. 95 (1994) 2505.
[2] A.G. Every and W. Sachse, Phys. Rev. B42 (1990) 8196.
[3] K.Y. Kim, A.G. Every and W. Sachse, Int. J. Med. Phys. B10 (1996) 235.
[4] J.F. Chai and T.T. Wu, J. Acoust. Soc. Am. 95 (1994) 3232.
[5] G.A. Northrop and J.P. Wolfe, Phonon Imaging: Theory and Applications. In Nonequilibrium Phonon Dynamics Ed. W.E. Bron, (Plenum, New York, 1985).
[6] A.G. Every, G.L. Koos and J.P. Wolfe, Phys. Rev. B29 (1984) 2190.
[7] W. Metzger and R.P. Huebener, Z. Phys. B73 (1988) 33.
[8] R.L. Weaver, M.R. Hauser and J.P. Wolfe, Z. Phys. B90 (1993) 27.
[9] K.U. Wuerz, J. Wesner, K. Hillmann and W. Grill, Z. Phys. B97 (1995) 487.
[10] R.E. Vines, S. Tamura and J.P. Wolfe, Phys. Rev. Lett. 74 (1995) 2729.